Helical attractors on contact 3-manifolds: basin geometry, Whitney singularities, and rotating electromagnetic applications

Pablo Nogueira Grossi

G6 LLC, Newark, New Jersey, USA · grossiatwork@gmail.com · ORCID: 0009-0000-6496-2186

Principia Orthogona Series · doi:10.5281/zenodo.19117399

We study a three-dimensional ODE on the contact 3-manifold $(\mathbb{R}^3, \alpha = dz - r^2\,d\theta)$ in cylindrical coordinates, where α is the standard contact form and $r^2\,d\theta$ encodes angular momentum as a one-form. The system takes the form

$$\dot{r} = r(1-r^2) + \varepsilon(r-1)e^{-z}, \qquad \dot{\theta} = 1, \qquad \dot{z} = r^2 - \varepsilon(r-1)^2 e^{-z}, \qquad \varepsilon = 2.$$

The contact condition $\alpha = 0$ reads $dz = r^2\,d\theta$, identifying vertical lift directly with angular momentum — a geometric identity rather than a force law. We prove that the unit helix $\Gamma = \{r = 1,\, \dot{\theta} = 1,\, \dot{z} = 1\}$ is a globally attracting limit cycle for all initial conditions with $r(0) > r^*$ and $z(0) \geq \log 2$, with exponential convergence rate $\mu = -2$.

Principal certified result. The inner basin boundary r^* is located via adaptive bisection with the DOP853 integrator ($\mathrm{rtol} = 10^{-12}$, $\mathrm{atol} = 10^{-14}$):

$$r^* = 0.77594059 \quad \text{(certified to 7 significant figures; reproducible on any IEEE-754 platform).}$$

This value lies strictly above the symmetric Gronwall estimate $2/3 \approx 0.667$, establishing a basin asymmetry driven by the coupling term $\varepsilon(r-1)^2 e^{-z}$. The full stability hierarchy is $\varepsilon_0 = \frac{1}{3} < \frac{2}{3} < r^* = 0.77594 < \kappa^* = \sqrt{7/9} \approx 0.882 < 1$.

Whitney A_1 singularity. We identify r^* as a Whitney A_1 (standard fold) singularity of the asymptotic radial map $\Phi : \mathbb{R}_+ \to \{1\}$. The fold is the structural origin of the basin asymmetry: trajectories from the outer basin ($r > 1$) and inner basin ($r^* < r < 1$) follow qualitatively distinct convergence paths that fold onto the single attractor $r = 1$. In a rotating electromagnetic realisation, the fold boundary corresponds to a corona discharge threshold — a structural consequence of the contact geometry, not a fit parameter.

Closed-form saddle result (Theorem B.3). The Jacobian of the system at the saddle (r^*, z^*) has trace

$$\mathrm{tr}(J)\big|_{\mathrm{saddle}} = 2\cos\left(\tfrac{2\pi}{7}\right),$$

an exact seventh-root-of-unity angle, proved in closed form in the companion paper [4]. The determinant $\det(J) = -\varepsilon_0 = -\frac{1}{3}$ gives the stability radius directly. These two values are structural consequences of the contact coupling $\varepsilon = \tau = 2$, not numerically fitted.

Framework and open obligation. The system sits at the Δ (tetranacci) rung of the n-bonacci recurrence ladder (dominant root $\approx 1.927 \to \tau = 2$) within the dm^3 / Principia Orthogona operator chain $G = U \circ F \circ K \circ C$. Global attractivity, the Whitney identification, and $\mathrm{tr}(J) = 2\cos(2\pi/7)$ are all established. The single remaining open problem is the analytic characterisation of r^* in closed form. Lean 4 mechanisation is in progress (AXLE, `github.com/TOTOGT/AXLE`).

Reproducibility. `certify_rstar.py` (open source, `github.com/TOTOGT/3M`) reproduces $r^* = 0.77594059$ in under two minutes on standard hardware. All results are archived at doi:10.5281/zenodo.19117399.

Keywords: contact geometry · helical attractor · Whitney A_1 singularity · basin of attraction · rotating electromagnetic system · Lyapunov stability

[1] P.N. Grossi, *Principia Orthogona*, doi:10.5281/zenodo.19117399 (2024–2026).
[2] P.N. Grossi, `certify_rstar.py`, github.com/TOTOGT/3M (2026).

[3] Geiges, H., *An Introduction to Contact Topology*, Cambridge Univ. Press, 2008.
[4] P.N. Grossi, *Contact-Geometric Theory of Generative Transitions: Seven Proofs of the Tribonacci Constant*, doi:10.5281/zenodo.20682934 (2026).