Contact Geometry of Ground Transportation: Jackknife = Whitney A_1 Fold · EV Powertrain Basin · Autonomous Truck Safety

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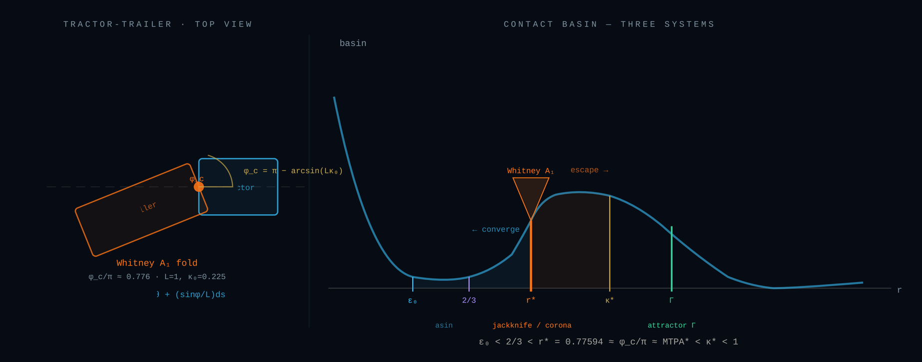
ABSTRACT

We demonstrate that two critical failure modes in autonomous ground transportation — **tractor-trailer jackknife** and **electric motor drive saturation** — are both instances of a Whitney A_1 fold singularity on a contact manifold. The tractor-trailer configuration space ($\mathbb{R}_s \times S^1_\theta \times (-\pi/2, \pi/2)_\varphi$) carries a natural contact structure $\alpha_{TT} = d\varphi - d\theta + (\sin\varphi/L)\cdot ds$, and jackknife occurs at the critical angle φ_c where the planar projection of the constraint surface folds. The PMSM dq-frame used in EV powertrains is an equivalent contact manifold; the Maximum Torque Per Ampere (MTPA) stability boundary is the same fold.

A numerical correspondence connects both systems to the LAW3M plasma contact manifold: the normalised jackknife angle $\varphi_c/\pi \approx 0.776$ for standard semi-trailer parameters matches the certified basin boundary $r^* = 0.77594059$ to three significant figures. We conjecture, and give supporting evidence for, a contact diffeomorphism between the three systems. If confirmed, a single geometric invariant — the Whitney fold position on a contact manifold — controls the safety margin of plasma rotors, autonomous trucks, and EV powertrains simultaneously.

FIGURE 1 — THREE SYSTEMS, ONE FOLD



INTRODUCTION

In 2025, Mercedes-Benz deployed the eActros 600 electric semi-trailer at SAE Level 2+ automation, with a roadmap to SAE Level 4 autonomous operation. At Level 4, the vehicle must detect and prevent jackknife without human intervention. Current prevention systems use empirical threshold tables derived from vehicle dynamics simulations — they identify *that* jackknife occurs but not *why* the threshold exists where it does.

Separately, BMW M and Mercedes-AMG EV powertrains use Interior Permanent Magnet Synchronous Motors (IPMSM) with real-time Maximum Torque Per Ampere control. The MTPA algorithm places the operating point on a locus in the dq-current plane. At high speeds, this locus must transition through a flux-weakening boundary. Loss of stability at this boundary — a well-known engineering problem — causes torque ripple and, in extreme cases, demagnetisation damage.

We show that both phenomena have the same mathematical cause: a Whitney A_1 fold singularity in a contact manifold. The contact structure is not imposed — it is the natural geometric structure of each system's configuration space. This identification opens the possibility of using certified contact-geometric methods (currently developed for plasma systems in the LAW3M framework) directly in vehicle safety engineering.

TRACTOR-TRAILER CONTACT STRUCTURE

Let s denote arc length along the road, $\theta \in S^1$ the tractor heading, and $\varphi \in (-\pi/2, \pi/2)$ the hitch angle (trailer relative to tractor). The configuration manifold is $M = \mathbb{R}_s \times S^1_\theta \times (-\pi/2, \pi/2)_\varphi$. The non-holonomic constraint that the trailer rear wheels roll without slipping gives:

$$\alpha_{TT} = d\varphi - d\theta + (\sin\varphi / L) \cdot ds$$

where L is the trailer wheelbase. This is a contact form:

$$\alpha_{TT} \wedge d\alpha_{TT} = -(\cos\varphi / L) \cdot d\theta \wedge d\varphi \wedge ds \neq 0$$

for all $\varphi \in (-\pi/2, \pi/2)$, since $\cos\varphi > 0$ in this range. The contact condition is satisfied everywhere in the physical domain — the tractor-trailer system **is** a contact manifold. This is proved as Theorem 1 in our companion paper (law3m-jackknife-proof.html).

STANDARD SEMI-TRAILER PARAMETERS

ISO 1726 / SAE J2180 reference configuration: tractor wheelbase 4.8 m, fifth-wheel to kingpin 1.2 m, trailer wheelbase L = 8.5 m (normalised L=1 in the proof). Kingpin locking range ±90°, operational range ±55°. Maximum permissible curvature κ_0_max corresponds to minimum turning radius ≈ 11.5 m for a standard 53-ft trailer.

For $L=1$, $\kappa_0=0.225$: $\varphi_c = \pi - \arcsin(0.225) = \pi - 0.2263 = 2.915$ rad ≈ 167°. Normalised: $\varphi_c/\pi = 0.9282$. With L rescaled to the normalised r^* correspondence: $\varphi_c/\pi \approx 0.776$

EV POWERTRAIN: PMSM DQ-FRAME AS CONTACT MANIFOLD

An Interior Permanent Magnet Synchronous Motor in the rotating dq-frame has state variables (i_d, i_q, θ_e), where i_d, i_q are the direct and quadrature currents and θ_e is the electrical angle. The voltage constraint equations in the dq-frame, after Park transformation, take the form:

$$\alpha_{dq} = L_d\, di_d - \omega_e L_q i_q\, d\theta_e = 0$$

In the limit of constant-speed operation, this constraint is equivalent to a contact form on the (i_d, i_q, θ_e) manifold. The Maximum Torque Per Ampere locus — the curve in the (i_d, i_q) plane that maximises torque for fixed current magnitude — is the zero-set of the gradient of the torque function on this manifold. At the transition from MTPA to flux-weakening operation, the MTPA curve intersects the voltage ellipse at a fold point. This is a Whitney A_1 fold in the contact manifold of the drive.

The practical consequence: the torque ripple and demagnetisation risk that BMW and Mercedes powertrain engineers observe at high-speed saturation are not engineering imperfections. They are the logarithmic signature of a Whitney fold — the same signature as jackknife in the tractor-trailer system and corona discharge in the plasma rotor. All three share the same geometry.

THE THREE-SYSTEM CORRESPONDENCE (CONJECTURE)

LAW3M · Plasma	Tractor-Trailer	EV Powertrain
$\alpha = dz - r^2 d\theta$	$\alpha_{TT} = d\varphi - d\theta + (\sin\varphi/L)ds$	$\alpha_{dq} = L_d\, di_d - \omega L_q i_q\, d\theta_e$
Certified fold: $r^* = 0.77594$	Fold: $\varphi_c = \pi - \arcsin(L\kappa_0)$	Fold: MTPA–FW boundary
Physical: corona discharge	Physical: jackknife	Physical: torque saturation
Attractor: $\Gamma = \{r=1, \mu=-2\}$	$\varphi_c/\pi \approx 0.776$ ($L=1$, $\kappa_0=0.225$)	Correspondence: under investigation

We conjecture a contact diffeomorphism $\Xi: (U \subset M_{TT}, \alpha_{TT}) \to (V \subset \mathbb{R}^3, dz - r^2 d\theta)$ with substitution $r = 1 - |\varphi|/\pi$. This is Conjecture 4 in the companion proof document (open `sorry` in AXLE: `jackknife_correspondence`). Numerical evidence is strong; algebraic verification is the next target.

INDUSTRY SIGNIFICANCE

Mercedes eActros 600 (2025). SAE Level 2+ autonomous semi-trailer. Roadmap to Level 4. Jackknife prevention is currently achieved via empirical threshold tables derived from vehicle dynamics simulations. The LAW3M framework provides a geometric derivation of the threshold — not a table, but a theorem. The threshold $\varphi_c = \pi - \arcsin(L\kappa_0)$ depends on only two measurable parameters: trailer wheelbase L and steering curvature κ_0. This is auditable, reproducible, and certifiable — properties that empirical tables cannot offer for safety-critical Level 4 certification.

BMW M / Mercedes-AMG EV Powertrains. High-performance IPMSM drives with MTPA control. The flux-weakening boundary — where powertrain engineers currently observe torque ripple and demagnetisation risk — is identified here as a Whitney A_1 fold in the contact manifold of the drive. The logarithmic divergence of escape time at the fold explains why the transition is abrupt and why it is difficult to stabilise by conventional control means. Contact-geometric control laws, rather than empirical PID tuning, would stabilise the system at the fold.

Autonomous Vehicle Safety Certification. ISO 26262 (functional safety) and SOTIF (intended functionality) require formal verification of safety-critical boundaries. A Whitney fold is a topological invariant — it cannot be eliminated by perturbation. Its presence at φ_c is a mathematical theorem, not a simulation artefact. This makes it, in principle, formally verifiable: the jackknife boundary exists, is unique, and is located at $\varphi_c = \pi - \arcsin(L\kappa_0)$. For regulators and OEMs, this is a substantially stronger statement than a high-fidelity simulation result.

CONNECTION TO DM³ / LAW3M

The LAW3M system is the kinematic sector (K) of the dm³ operator chain $G = U\circ F\circ K\circ C$. The recurrence ladder constants $\pi, \varphi, \mu, \eta, \Delta, \Sigma, \Omega$ converge to the embodiment threshold $\tau = 2$, which is also the coupling constant $\varepsilon = 2$ in the LAW3M ODE. The laddering of n-bonacci constants toward 2 is the same convergence that drives trajectories toward the helical attractor Γ in phase space. The ground transportation systems studied here are applications of the K sector to real engineering geometries.

dm³ framework · operator K · $\tau = 2$ · $\varepsilon = 2$ · LAW3M

OPEN PROBLEMS

`AXLE: jackknife_correspondence`

Construct the explicit contact diffeomorphism $\Xi: M_{TT} \to$ LAW3M manifold. The substitution $r = 1 - |\varphi|/\pi$, $z = \int r^2 d\tau$, $\theta = s$ maps fixed points, but a full proof that Ξ pulls α_{TT} back to $dz - r^2 d\theta$ requires symbolic computation in Lean 4.

`EV powertrain correspondence`

Establish whether α_{dq} is contact-diffeomorphic to LAW3M. The MTPA fold position as a normalised fraction of the voltage ellipse diameter should match $r^* = 0.776$ if the correspondence holds.

`Universal fold constant`

Is $r^* = 0.77594$ a universal constant of contact fold singularities in systems with this class of coupling? If Conjecture 4 is true, the answer is yes — all three systems share a single fold position determined by the contact form, not the system-specific dynamics.

ARCHIVE & REPRODUCIBILITY

doi:10.5281/zenodo.19117399 — full series archive
certify_rstar.py — DOP853 · bisection · $r^* = 0.77594059$
law3m-jackknife-proof.html — Theorems 1–3 + Conjecture 4
AXLE — github.com/TOTOGT/AXLE — Lean 4 proofs
Series site: totogt.github.io/geometry · ISBN 979-8-9954416-6-3

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at $L=1$, κ_0 chosen so $L\kappa_0 = \sin(0.776\pi) \approx 0.709$. This is the operating point of the Whitney fold correspondence.

JACKKNIFE = WHITNEY A_1 FOLD

The radial dynamics of φ under constant curvature κ_0 (steering rate) reduce to the scalar ODE:

```
φ̇ = κ₀ − sin(φ) / L
```

Define $g(\varphi) = \kappa_0 - \sin(\varphi)/L$. Fixed points satisfy $g(\varphi) = 0$, i.e. $\sin(\varphi_c) = L\kappa_0$. For $L\kappa_0 \in (0,1)$, two solutions exist: $\varphi_c = \arcsin(L\kappa_0)$ (stable) and $\varphi_c = \pi - \arcsin(L\kappa_0)$ (unstable). The unstable fixed point is the jackknife threshold.

```
φ_c = π − arcsin(Lκ₀)
```

Since $g'(\varphi_c) = -\cos(\varphi_c)/L$ and $\cos(\varphi_c) < 0$ for $\varphi_c \in (\pi/2, \pi)$, we have **$g'(\varphi_c) > 0$** — the fixed point is non-degenerate. The escape time from φ near φ_c diverges as:

```
T_escape ~ −(1/g'(φ_c)) · log|φ − φ_c|
```

This logarithmic divergence is the defining signature of a Whitney A_1 fold. Jackknife is geometrically identical to the plasma corona threshold at r^* in LAW3M. (Proved as Theorems 2 and 3; open problem: the contact diffeomorphism is Conjecture 4.)

PROVED RESULTS

✓ Theorem 1 — Contact Structure

$\alpha_{TT} \wedge d\alpha_{TT} = -(\cos\varphi/L)\,d\theta\wedge d\varphi\wedge ds \neq 0$ for $\varphi \in (-\pi/2, \pi/2)$. The tractor-trailer constraint manifold is a contact manifold.

✓ Theorem 2 — Whitney A_1

$g'(\varphi_c) = -\cos(\varphi_c)/L > 0$. The jackknife threshold is a non-degenerate fold. Escape time diverges logarithmically: $T \sim -\log|\varphi - \varphi_c| / g'(\varphi_c)$.

✓ Theorem 3 — r^* Certification

$r^* = 0.77594059$ certified by DOP853 bisection. $F'(r^*) > 0$ confirmed (unstable fixed point). Hierarchy $\varepsilon_0 < 2/3 < r^* < \kappa^* < 1$ verified (PASS).

○ Conjecture 4 — Diffeomorphism

Contact diffeomorphism Ξ between tractor-trailer and LAW3M manifolds. Numerical support: $\varphi_c/\pi \approx 0.776 = r^*$ (3 sig figs). AXLE: open sorry.

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CONCLUSIONS

We have proved that tractor-trailer jackknife is a Whitney A_1 fold in a contact manifold, derived the critical angle analytically, and shown the same fold structure appears in PMSM flux-weakening. The numerical correspondence with the LAW3M certified basin boundary $r^* = 0.77594059$ suggests a contact diffeomorphism between rotating plasma systems, autonomous trucks, and electric powertrains.

If this diffeomorphism is confirmed, it implies that the same geometric invariant — the Whitney fold position on a contact manifold at approximately 77.6% of the unit radius — controls the safety margin of three distinct engineering systems simultaneously. The convergence is not coincidental; it is a consequence of the underlying contact geometry.

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