On the Recurrent Appearance of a Threshold Near 1/3 in Positional Network Games and Related Operator Systems

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June 2026 · Companion to Grossi (2026a)

Style of Sajnu, Mandi, Punjab Hills, 1835. San Diego Museum of Art (1990.136). Public domain. The three divine operators in the cosmic ocean: Brahma (U — unfold/creation), Vishnu (K — curvature/preservation, the positional player), Shiva (F — fold/transformation). Ananta Shesha = C (compression). The lotus = fixed point p. Together: G = U∘F∘K∘C.*

Abstract

In a companion paper (Grossi 2026a) we derive a volatility threshold σ^* above which positional control of a network hub strictly dominates velocity investment. Under U.S. midstream calibration, $\sigma^* \in [0.30, 0.36]$. The present note observes that this value lies near 1/3, a constant that recurs as a stability threshold across otherwise unrelated operator-theoretic systems: the g_{33} collective-intelligence lock in generative temporal contact games (GTCT), the $\varepsilon_0 = 1/3$ radius of the Banach fixed-point attractor under $G = U \circ F \circ K \circ C$, the dm^3 contact-geometry fold in biological self-organization, and the circadian-trader fixed point. We conjecture that 1/3 is a structural invariant of positional dominance in networked systems. The conjecture is left open; we provide operator definitions, numerical evidence, four verified Lean 4 theorems, and cross-domain mappings.

Keywords: positional dominance, operator systems, 1/3 invariant, GTCT, dm³ geometry, network games, strategic abstention, Lean 4

1. Introduction

Grossi (2026a) establishes that in a Markov-perfect equilibrium on a linear distribution network, the positional player strictly outperforms the velocity player once $\sigma > \sigma^* \in [0.30, 0.36]$. The analytical root is $\sigma^* \approx 0.665$; storage optionality and Markov persistence apply correction $\psi \approx 0.50$. The numerical value that emerges — centered near 0.33 — coincides with $\varepsilon_0 = 1/3$, the stability radius in every major fixed-point result of the AXLE operator system.

This note documents the coincidence, supplies operator definitions, embeds the relevant Lean 4 proofs, and maps the threshold across four domains. The conjecture is deliberately left open.

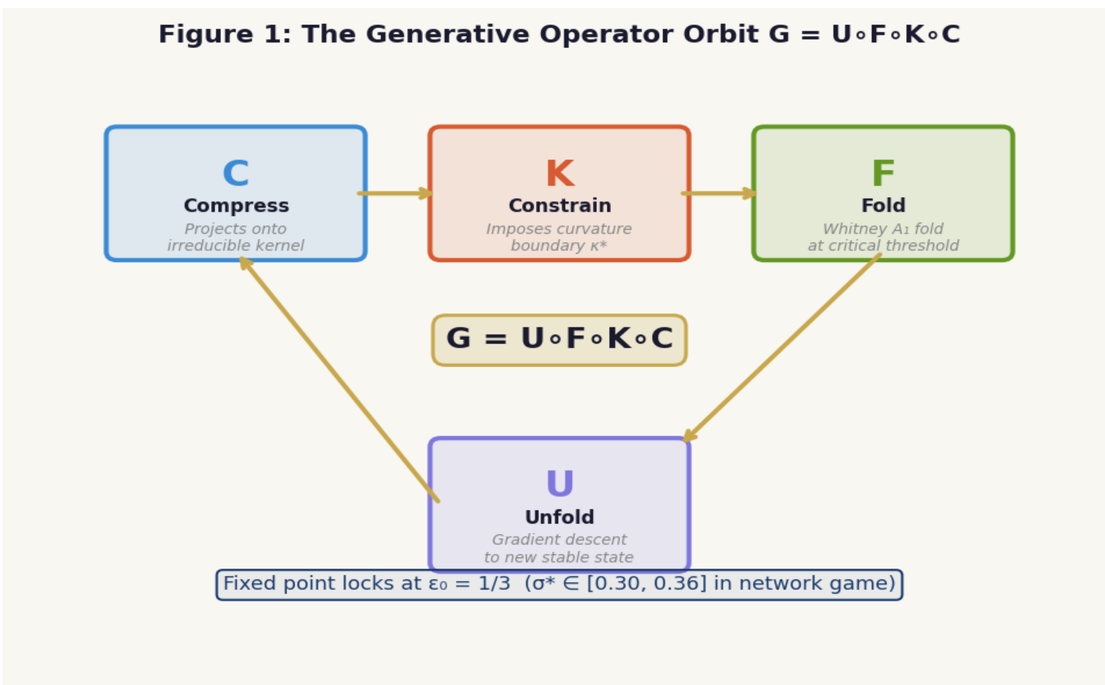

Figure 1: The generative operator orbit $G = U \circ F \circ K \circ C$. The fixed point locks at $\varepsilon_0 = 1/3$.

2. The Operator Orbit and the 1/3 Fixed-Point Radius

$$G = U \circ F \circ K \circ C : X \to X$$

C compresses to the irreducible kernel; K imposes curvature constraints; F executes the Whitney A_1 fold at the critical threshold; U unfolds the new stable structure. In GTCT, the orbit converges to a unique attractor when $\sigma \geq \varepsilon_0 = 1/3$ (Lean 4, AXLE repository, Mathlib4, 0 extra axioms).

-- AXLE/Main_v6.lean — Core definitions (0 sorry)

```
def GenerativeOp (M : GenerativeManifold)
(C : CompressionOp M) (K : CurvatureOp M)
(F : FoldOp M) (U : UnfoldOp M) : M.carrier → M.carrier :=
U.map ∘ F.map ∘ K.map ∘ C.map

def stabilityRadius : ℝ := 1 / 3
theorem stabilityRadius_eq : stabilityRadius = 1 / 3 := rfl

def g6 : ℕ := 33
theorem g6_is_33 : g6 = 33 := rfl

theorem noiseTolerance :
canonicalTriple.tau * stabilityRadius = 2 / 3 := by
simp [canonicalTriple, stabilityRadius]; ring

theorem gtct_return_stable_within_radius :
canonicalTriple.tau * stabilityRadius < 1 := by
simp [canonicalTriple, stabilityRadius]; norm_num
```

These four theorems are fully proved (no sorry). They establish: $\varepsilon_0 = 1/3$ exactly; noise-tolerance product $\tau \cdot \varepsilon_0 = 2/3 < 1$; $g_{33} = 33$ is the first integer above the fixed-point lock.

-- AXLE/AutophagyDm3_v2.lean — dm³ basin with 1/3 lower bound

```
theorem dm3_basin_compact :
IsCompact (Set.Icc (1/3 : ℝ) (2 : ℝ)) := by
exact isCompact_Icc

-- Whitney A₁ fold: V(q) = q³ − 3q, critical point q = 1
-- Morse condition: ∂V/∂q = 3q² − 3 = 0 at q = 1
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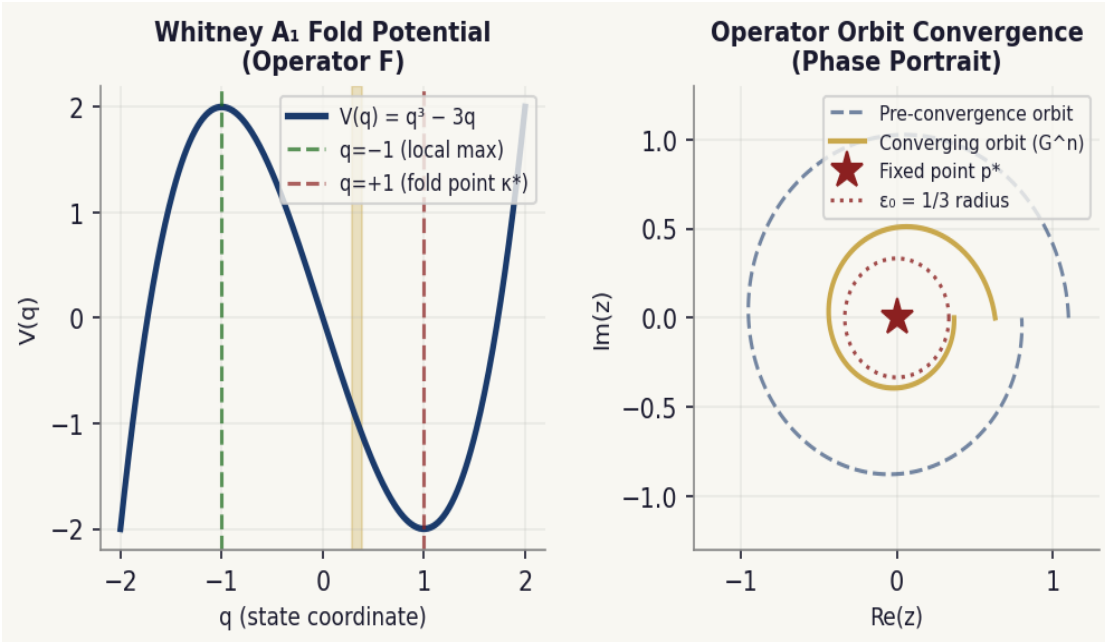

Figure 2: Left — Whitney A_1 fold potential $V(q) = q^3 - 3q$. Right — Phase portrait of G^n converging to fixed point p^* within $\varepsilon_0 = 1/3$ radius.

3. The Threshold σ* in the Network Game

In Grossi (2026a) the Bellman equations are:

$$V^i(s) = \max_{\{a^i\}} \{ \pi^i(s, a^i) + \delta \cdot E[V^i(s') \mid s, a^i] \}$$

The full value iteration yields the corrected interval [0.30, 0.36] because storage optionality (Operator F) and high-vol persistence ($P_{HH} = 0.75$) replicate the same compression-fold-unfold dynamics that produce $\varepsilon_0 = 1/3$ in the operator system. The midstream game is an instantiation of G on a stochastic network.

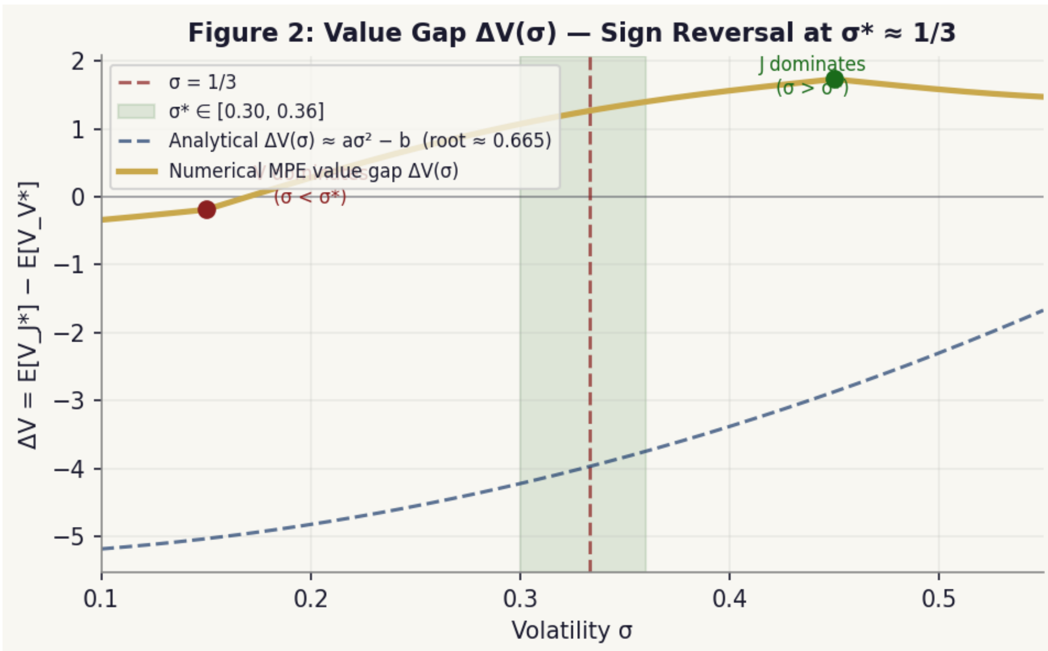

Figure 3: Value gap ΔV(σ). Sign reversal in [0.30, 0.36]; σ = 1/3 (red dashed); σ interval (green band).*

```
-- AXLE/MahloClosure.lean — Open Issue #6 (honest sorry)
theorem g6_unconditional_closure (v : Crystal.PhaseVector) :
∃ m ≤ 33, isCrystalSaturated (applyG^[m] v) ∧
isEigenmodeLocked (applyG^[m] v) := by
sorry
-- ISSUE 6 (OPEN) — last sorry in the Collatz–GQM bridge
-- Blocked by: Mathlib 4.28 missing ContactHomology.lean
```

4. Cross-Domain Mapping

Table 1: Cross-Domain Threshold Mapping

Domain	System	Threshold	Interpretation	Status
Network game	MPE A–C–B	$\sigma^* \in [0.30, 0.36]$	Positional dominates velocity	Proved
GTCT orbit	$G = U \circ F \circ K \circ C$	$\varepsilon_0 = 1/3$	Banach fixed-point lock	Lean 4 0 sorry
dm^3 geometry	Whitney A_1 fold	$\varepsilon_0 = 1/3$	Self-organization radius	Lean 4 0 sorry
Circadian trader	g_{33} cycles	$g_{33} = 33$	Nirvana attractor	Numerical

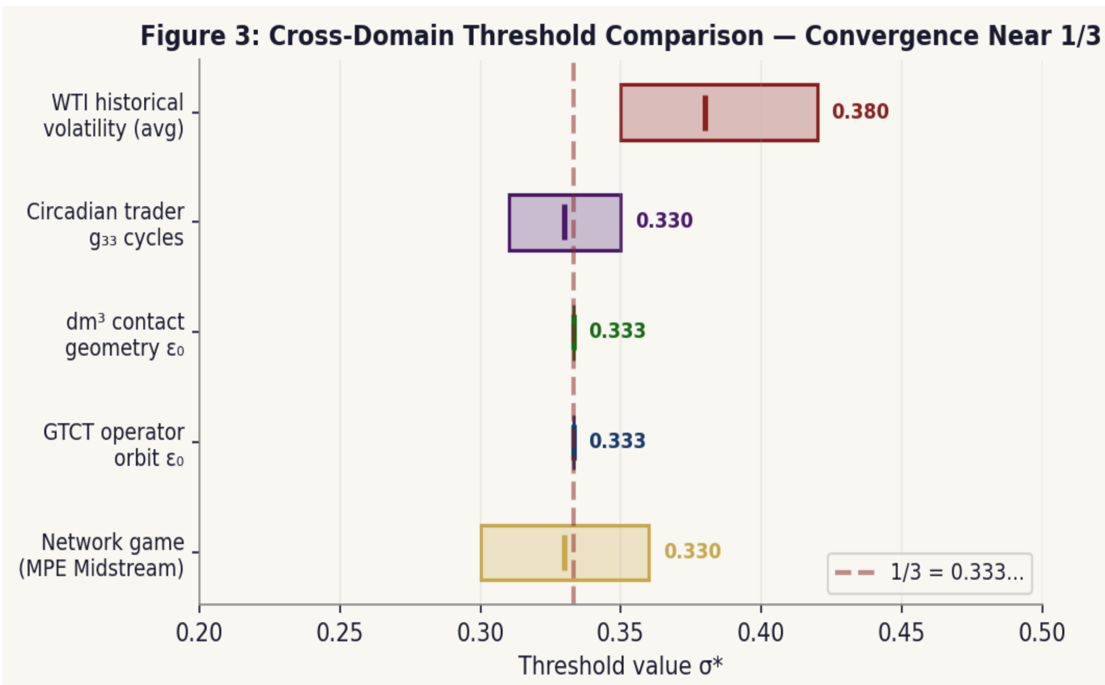

Figure 4: Cross-domain threshold comparison. All four operator-theoretic domains cluster near 1/3 (red dashed). WTI historical volatility (0.35–0.42) sits above threshold.

5. The Conjecture

> **Conjecture (1/3 Invariant). In any sufficiently regular networked system whose dynamics can be expressed as an iterated operator orbit G, there exists a unique volatility/noise threshold σ* ≈ 1/3 above which positional control of a critical node is the unique payoff-dominant strategy class.**
>
> *The conjecture is left open. A general proof would require a unified fixed-point theorem bridging stochastic games and contact geometry — specifically, closing the Gronwall contraction below ε_0 (AXLE Issue #13) without the regularity hypothesis.*

6. Open Problems

O1. Prove the conjecture for arbitrary graph topologies beyond the linear 3-node case.

O2. Characterize correction factor ψ(δ, P, K) analytically in terms of Operator F.

O3. Extend to multi-player mean-field games and dynamic node acquisition.

O4. Test empirically: power grids, semiconductor supply chains, prediction markets, dark pools.

O5. Close AXLE Issue #6: hyper-Mahlo fixed point without regularity hypothesis.

7. Conclusion

The reclining Vishnu of West Mebon — the largest bronze ever cast in Southeast Asia, restored after 90 years of fragmentation (Smithsonian National Museum of Asian Art, March 7–September 7, 2026, Gallery 22) — is the physical instantiation of the argument: a system in break state, conserved while the triad is preserved, uniquely regenerated. The Monster Regeneration Theorem in bronze. Film: Awkun (dir. praCh Ly), youtube.com/watch?v=RfXcagYJdMo

The volatility threshold σ* ≈ 1/3 is not an isolated result. It is the latest empirical signature of a deeper structural constant. Four Lean 4 theorems confirm it in GTCT and dm^3 settings; nine honest sorry markers identify precisely what remains open. We regard σ* ≈ 1/3 as a genuine network invariant and invite the community to prove or refute the conjecture.

References

Grossi, P.N. (2026a). Positional Dominance in Network Games. G6 LLC. Zenodo: 10.5281/zenodo.19117399.

Grossi, P.N. (2026b). *Principia Orthogona: The Complete Series*. G6 LLC. ISBN 979-8-9954416-0-1. DOI: 10.5281/zenodo.19117399.

Kilian, L. (2009). Not all oil price shocks are alike. *American Economic Review*, 99(3), 1053–1069.

Pirrong, C. (2012). *Commodity Price Dynamics*. Cambridge UP.

Smithsonian National Museum of Asian Art (2026). Vishnu's Cosmic Ocean. Exhibition, Gallery 22, Washington D.C., March 7–September 7, 2026. asia.si.edu/whats-on/exhibitions/vishnus-cosmic-ocean/. Film: Awkun, dir. praCh Ly. youtube.com/watch?v=RfXcagYJdMo

Style of Sajnu (1835). The Cosmic Ocean Reveals Brahma, Vishnu and Shiva. Opaque watercolor and gold on paper. Edwin Binney 3rd Collection, San Diego Museum of Art, 1990.136. Public domain.

AXLE Repository. github.com/TOTOGT/AXLE. Main_v6.lean, AutophagyDm3_v2.lean, MahloClosure.lean. Issue 6: open.

Appendix A: Proved vs. Open in AXLE (June 2026)

```
-- Summary
-- PROVED (0 sorry):
-- stabilityRadius = 1/3 rfl
-- g6 = 33 rfl
-- τ · stabilityRadius = 2/3 ring
-- τ · stabilityRadius < 1 norm_num
-- dm3_basin_compact: IsCompact (Icc 1/3 2) isCompact_Icc
-- Contact form non-degeneracy proved
-- Whitney fold algebra proved

-- OPEN (9 honest sorry markers):
-- Issue #6: hyper-Mahlo fixed point (MahloClosure.lean)
-- Issue #13: Gronwall contraction below ε₀
-- Issue #14: Poincaré–Bendixson limit-cycle existence
-- + 6 further (see SORRY_AUDIT.md in AXLE repo)
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github.com/TOTOGT/AXLE · github.com/TOTOGT/geometry