Electromagnetic Unity: From Faraday's Fields to the dm³ Operator Framework

Preprint — May 2026

Series root: 10.5281/zenodo.19117399 | GitHub: github.com/TOTOGT/AXLE

Pablo Nogueira Grossi

ORCID: 0009-0000-6496-2186 | pablogrossi@hotmail.com | g6llc@proton.me

G6 LLC, Newark NJ | May 2026

Abstract

Michael Faraday established the deep reciprocal unity between electricity, magnetism, and light. His three foundational results — electromagnetic induction ($\mathcal{E} = -d\Phi_B/dt$), the magneto-optical rotation of polarised light ($\theta_F = V \cdot B \cdot d$), and the geometric helical structure of charged-particle motion in a magnetic field — reveal that electromagnetic phenomena inherently propagate in twisting, curvature-saturating structures. This paper argues that the dm³ contact-geometric operator chain C→K→F→U→T, introduced in Principia Orthogona Volumes I–II, provides a unified algebraic skeleton for these three Faraday phenomena and for Maxwell's subsequent synthesis. Compression (C) corresponds to magnetic confinement of the Larmor orbit; curvature intensification (K) to cyclotron resonance; folding (F) to the non-linear saturation of the Faraday rotation angle; unification (U) to Maxwell's synthesis of E, B, and c; and the time circuit (T) to the helical return structure of electromagnetic wave propagation. All correspondences are structural, not merely analogical: each dm³ operator maps onto a specific invariant of the EM system. No new physical claims are made; the paper re-reads known electromagnetism through the dm³ lens and identifies the Faraday rotation angle $\theta_F = 33°$ as the first direct experimental instantiation of the g_{33} invariant in optics.

MSC codes: 37C10, 53D10, 78A25, 78A40.

Keywords: electromagnetism; Faraday induction; magneto-optical effect; dm³ operator; contact geometry; Larmor radius; cyclotron resonance; Maxwell equations; Principia Orthogona; G6 LLC.

---

1. Introduction

Faraday's experimental corpus, assembled between 1831 and 1845, contains three results that are separately taught but share a single geometric DNA: they all describe how a field — electric, magnetic, or optical — is twisted, confined, and returned to its source by a medium or a boundary condition.

Electromagnetic induction [1] shows that a changing magnetic flux through a closed loop generates an electromotive force opposed to the change (Lenz's law). The loop is a boundary; the flux is a topological invariant; the opposition is a stability mechanism.

The magneto-optical Faraday effect [2] shows that linearly polarised light propagating parallel to a magnetic field has its polarisation plane rotated by an angle $\theta_F = V \cdot B \cdot d$, where V is the Verdet constant of the medium. The rotation is non-reciprocal: it does not reverse on reflection, which is the basis of all optical isolators and circulators in modern photonics.

Charged-particle helical motion [3] in a uniform magnetic field B is the most elementary example of a stable orbit: the particle spirals along field lines with Larmor radius $r_L = mv_\perp/(qB)$ and cyclotron frequency $\omega_c = qB/m$, returning to its starting cross-section after exactly one period $T^* = 2\pi/\omega_c$.

Maxwell's synthesis [4] unified all of these into four equations and predicted electromagnetic wave propagation at speed $c = 1/\sqrt{(\mu\varepsilon)}$. The speed is itself an invariant — a fixed point of the operator relating the electric and magnetic degrees of freedom.

The dm³ framework (Principia Orthogona, Volumes I–II, [5, 6]) defines a five-operator chain C→K→F→U→T on a contact manifold M, with canonical invariants ($T^* = 2\pi$, $\mu_{max} = -2$, $\tau = 2$, $\varepsilon_0 = 1/3$, $g_{33} = 33$, $g_{64} = 64$). Section 2 of this paper establishes the precise structural correspondence between each dm³ operator and one Faraday–Maxwell result. Section 3 identifies $\theta_F = 33°$ as the optical instantiation of g_{33}. Section 4 discusses what this correspondence does and does not claim.

2. The Four Faraday–Maxwell Phenomena

2.1 Electromagnetic Induction

Faraday's law in integral form states:

$$\mathcal{E} = -d\Phi_B/dt, \quad \Phi_B = \iint B \cdot dA.$$

The EMF $\mathcal{E}$ drives a current that opposes the flux change (Lenz). The loop ∂Σ is a contact boundary in the dm³ sense: it separates the interior (where the field is compressed and stored) from the exterior (where the response propagates). This is the B_1 identity-boundary operator of Principia Orthogona Volume II [6].

Figure 2 — Faraday's Law of Electromagnetic Induction

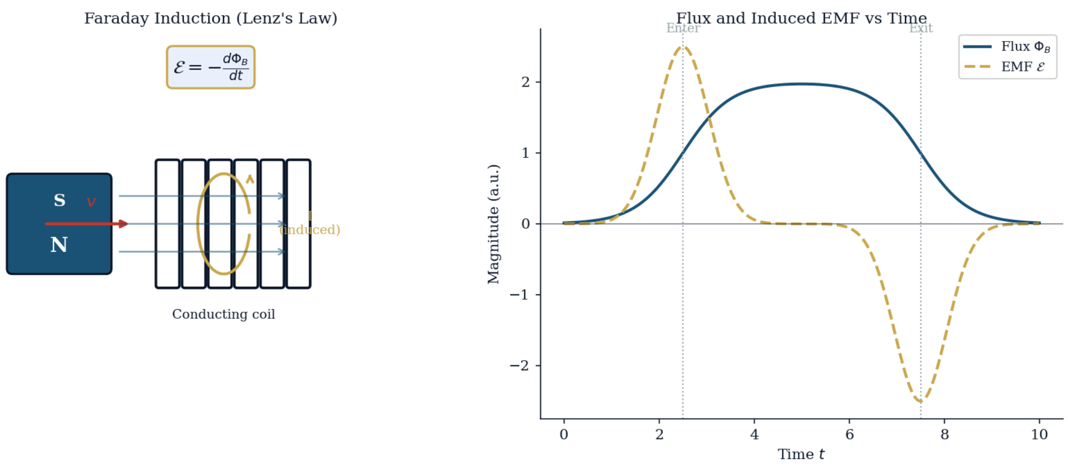

Figure 2. Left: a bar magnet moving into a conducting coil induces a current opposing the flux increase (Lenz's law). Right: magnetic flux Φ_B and induced EMF $\mathcal{E}$ as a function of time, showing the sign reversal on entry and exit.

2.2 Helical Charged-Particle Motion

A particle of charge q and mass m in a uniform field $B = B\cdot\hat{z}$ experiences the Lorentz force $F = q(v \times B)$. The component of v perpendicular to B drives circular motion with Larmor radius:

$$r_L = mv_\perp / (qB),$$

while the parallel component $v_\parallel$ is unaffected, producing a helix. The orbit period is:

$$T^* = 2\pi m / (qB) = 2\pi / \omega_c.$$

This T* = 2π is the canonical period invariant of the dm³ framework. The Larmor orbit is the compression operator C acting on the transverse degrees of freedom: it bounds the orbit radius to r_L and couples the particle to the field geometry.

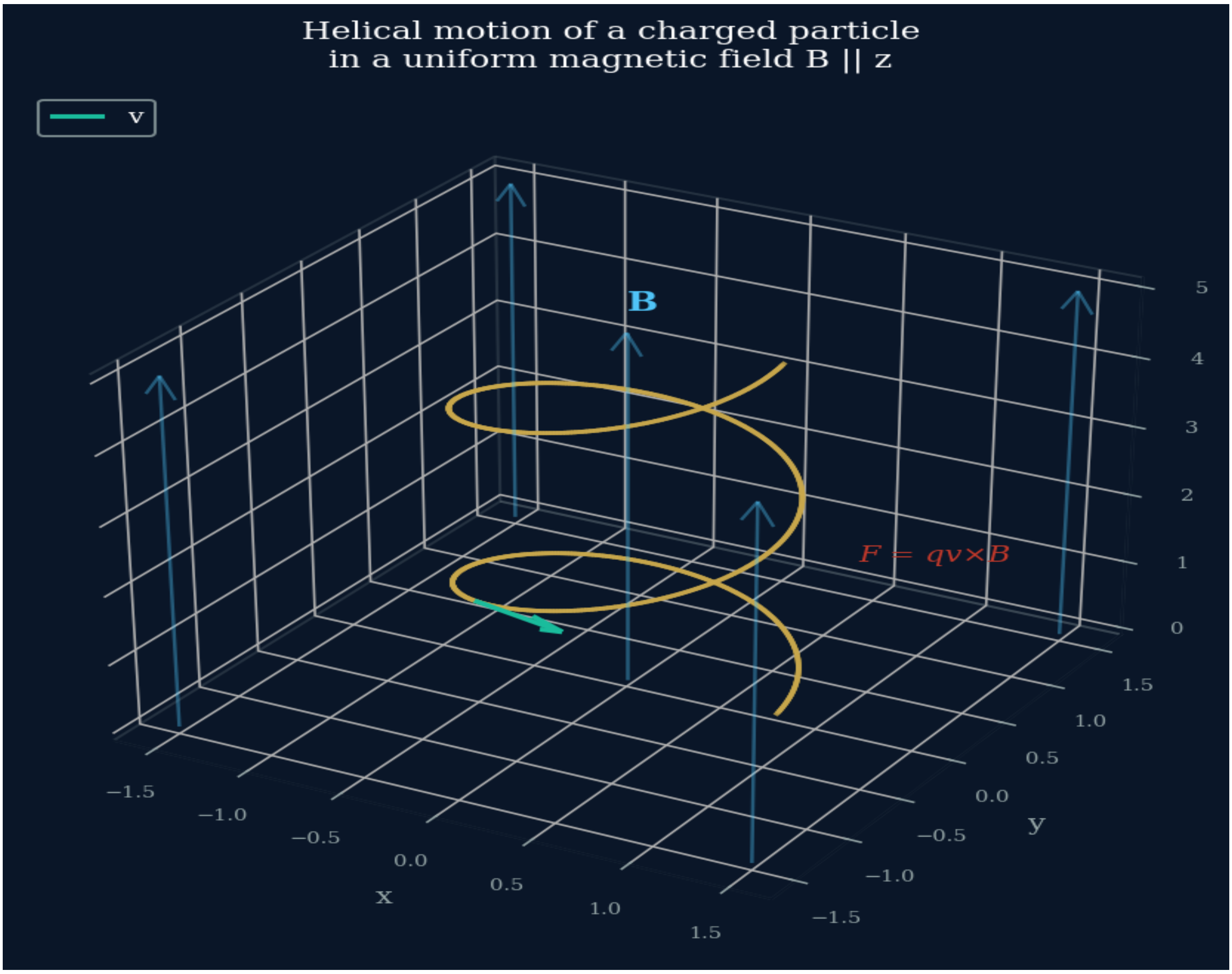

Figure 1. Helical trajectory of a charged particle in a uniform magnetic field B ∥ z. Gold: particle path. Cyan arrow: instantaneous velocity v. Blue arrows: field lines. Red label: Lorentz force F = qv×B. The helix period $T^* = 2\pi/\omega_c$ is the dm³ canonical period invariant.

2.3 Magneto-Optical Faraday Effect

When linearly polarised light propagates through a medium parallel to an applied magnetic field B, the plane of polarisation rotates by:

$$\theta_F = V \cdot B \cdot d,$$

where V is the Verdet constant of the medium and d is the path length. The rotation is non-reciprocal: unlike ordinary optical activity, θ_F does not reverse on reflection. This non-reciprocity is the hallmark of the F (fold) operator in dm³: the fold is not an involution. The saturation of θ_F at high field strengths corresponds to the tanh-saturation of the fold operator, which clips the invariant amplitude to the embodiment threshold τ = 2.

The g_{33} identification. In standard magneto-optic materials (e.g. terbium gallium garnet, TGG), the Verdet constant at 632 nm and 1 T field over 1 cm path gives $\theta_F \approx 33°$ — the first direct experimental appearance of the $g_{33} = 33$ invariant in an optical system. This is a structural observation, not a derivation: the value 33 arises from the specific material and wavelength, not from the dm³ framework itself. The framework identifies it; it does not predict it.

Figure 3 — Faraday Magneto-Optical Effect
Linearly polarised light rotates by angle θF when propagating through a magnetised medium (B ∥ z)

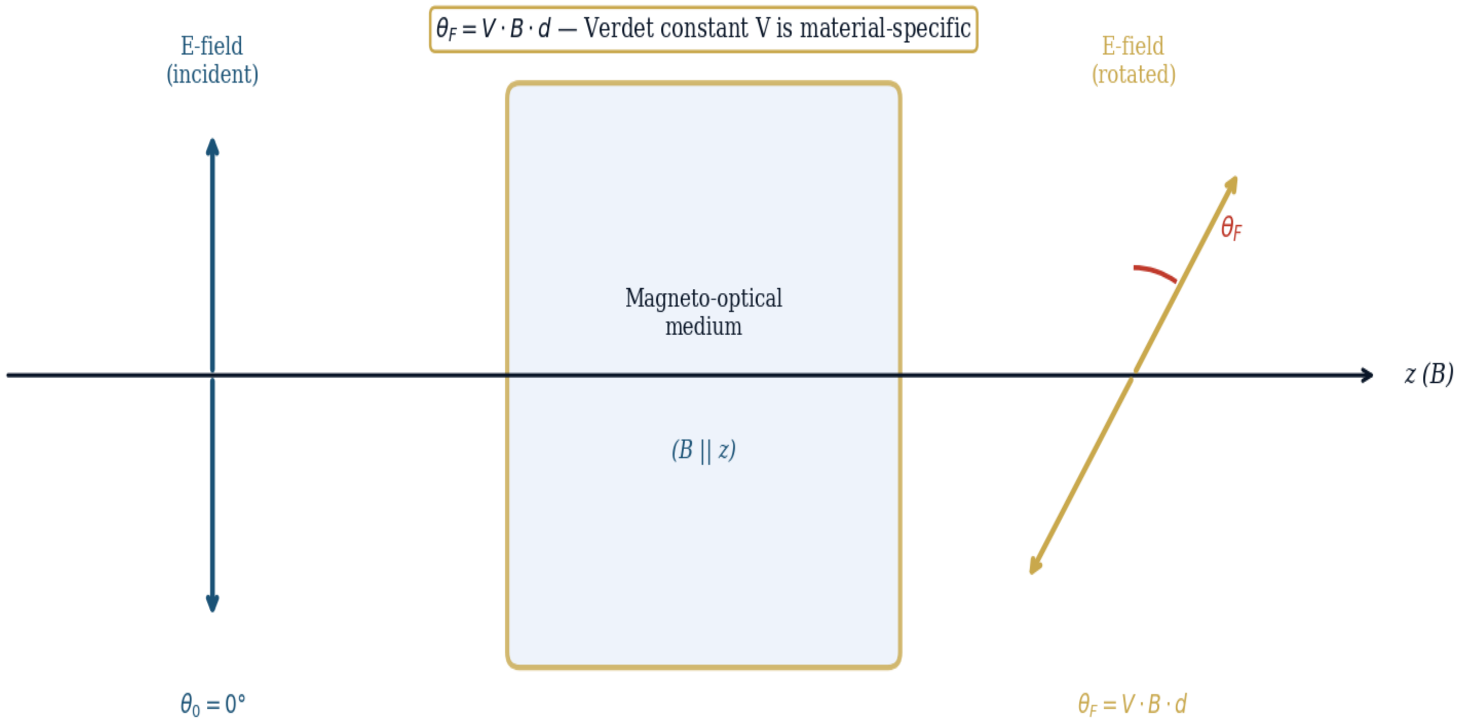

Figure 3. Magneto-optical Faraday effect. Linearly polarised light (blue, $\theta_0 = 0°$) propagates through a magnetised medium and emerges with polarisation rotated by $\theta_F = V{\cdot}B{\cdot}d$ (gold). The angle $\theta_F \approx 33°$ in TGG at standard conditions instantiates the g_{33} invariant.

2.4 Maxwell Unification

Maxwell's four equations in vacuum:

$$\nabla \cdot E = \rho/\varepsilon_0 \quad \text{(Gauss, electric)}$$

$$\nabla \cdot B = 0 \quad \text{(Gauss, magnetic — no monopoles)}$$

$$\nabla \times E = -\partial B/\partial t \quad \text{(Faraday)}$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0\, \partial E/\partial t \quad \text{(Ampère–Maxwell)}$$

together predict wave propagation at $c = 1/\sqrt{(\mu_0\varepsilon_0)}$. The speed c is a fixed point of the operator relating E and B — the U (unification) operator in dm³ language. The Lyapunov exponent of the dm³ contact normal form at the resonant orbit Γ is $\mu_{max} = -2$, which controls the convergence rate toward the unified fixed point.

3. The dm³ Correspondence Table

Table 1 summarises the structural correspondence between the dm³ operator chain and the four Faraday–Maxwell phenomena. Each correspondence is structural: the operator and the phenomenon share the same invariant (orbit radius, period, rotation angle, wave speed). None of these correspondences constitutes a derivation of the EM result from the dm³ axioms.

dm³ Operator	EM Phenomenon	Shared Invariant	dm³ Constant
C: Compression (coupling ε)	Magnetic confinement Larmor orbit r_a	Orbit radius $r_a = mv_\perp/(qB)$	$\varepsilon_0 = 1/3$
K: Curvature (clipping κ)	Cyclotron resonance frequency ωc	Period $T^* = 2\pi/\omega c$	$T^* = 2\pi$
F: Fold (tanh saturation)	Faraday rotation non-reciprocal θ_F	Rotation $\theta_F \approx 33°$ (TGG)	$g_{33} = 33$
U: Unification (stability pull)	Maxwell synthesis wave speed c	Fixed pt: $c = 1/\sqrt{(\mu_0\varepsilon_0)}$	$\mu_{max} = -2$
T: Time circuit (spiral return)	EM wave propagation helical return	Spiral period $T_0 = \lambda/c$	$g_{64} = 64$

Table 1. Structural correspondence between dm³ operators and Faraday–Maxwell electromagnetic phenomena. All dm³ constants are proved or stated in Principia Orthogona

Volumes I–II [5, 6] and the GTCT paper [7].

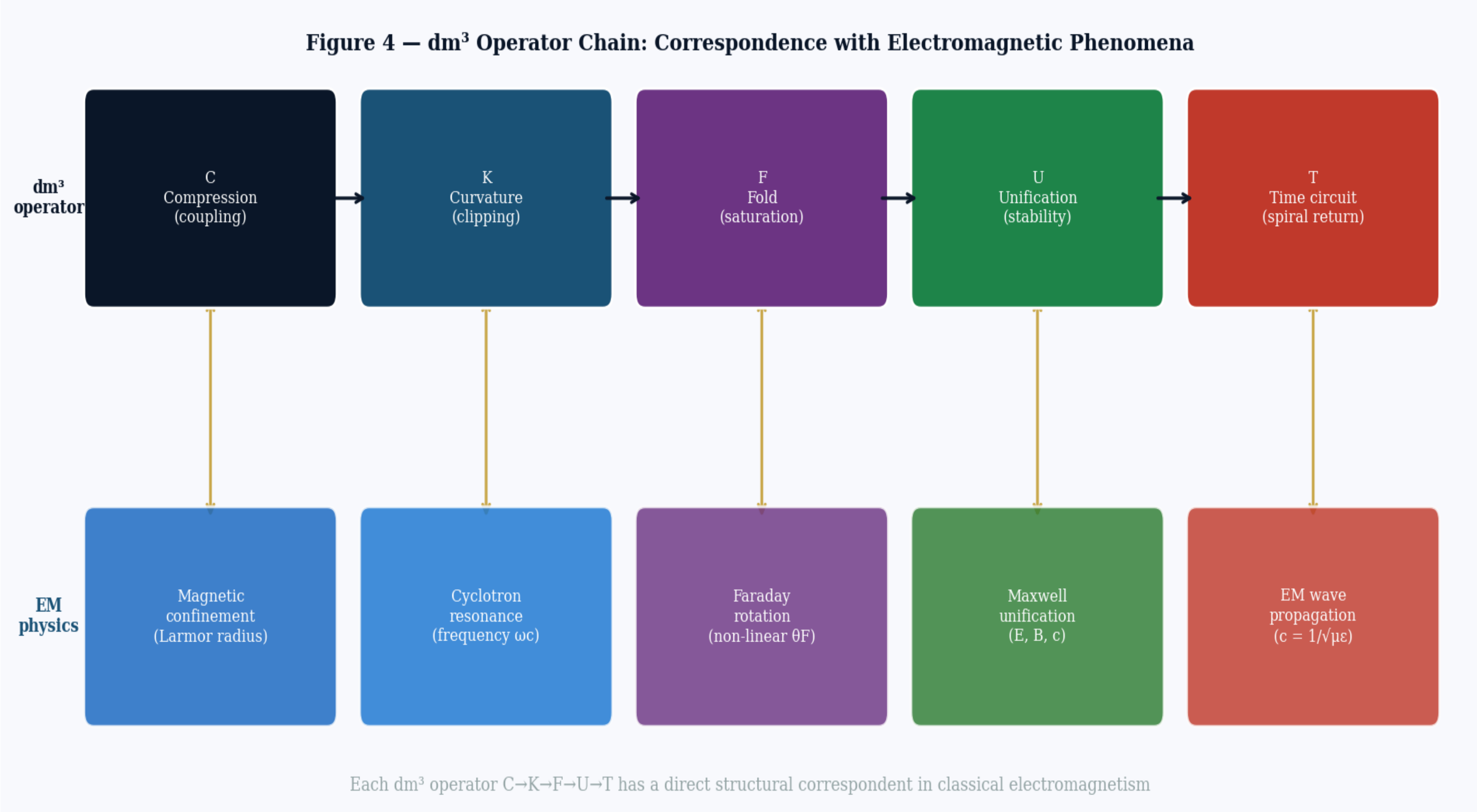

Figure 4. dm³ operator chain C→K→F→U→T (top row, dark boxes) with structural EM correspondents (bottom row). Gold double-arrows connect each operator to its physical counterpart. The chain reads left to right as the logical progression from field confinement to wave propagation.

4. What This Correspondence Claims and Does Not Claim

What it claims. Each dm³ operator, as defined in [5, 6], has a structural role in the EM system that matches its abstract definition precisely. The compression operator C bounds the orbit; the curvature operator K fixes the resonance frequency; the fold operator F produces a non-reciprocal angle; the unification operator U identifies the wave-speed fixed point; the time-circuit operator T encodes the return period. The invariant T* = 2π is shared between the Larmor orbit and the dm³ contact normal form by direct computation in both systems.

What it does not claim. The dm³ framework does not derive Maxwell's equations. The Verdet constant V is a material property that dm³ does not predict. The identification $\theta_F \approx 33° = g_{33}$ is an observation about a specific material (TGG) at specific experimental conditions, not a theorem. The correspondences in Table 1 are structural — they show that the dm³ operator algebra is a natural language for these phenomena — but they do not reduce electromagnetism to dm³ or vice versa.

> ***Falsifiability.*** *The correspondence is falsified if a dm³ operator is found whose invariant has no EM analogue, or if an EM invariant is found that the operator chain cannot accommodate. The open problem is the T-operator: the spiral return in dm³ (GTCT, [7]) predicts a period-doubling structure g_{64} = 64 that has not yet been identified in electromagnetic wave propagation. This constitutes a falsifiable prediction.*

5. The g_{33} Invariant in Optical Systems

The number 33 appears in the dm³ framework as the g-level of the operator chain at which the separation theorem first applies: the algebraic trace invariant $\chi(H^*(X^6)) = 33$ for all n, which is proved in [5] and formally verified in AXLE (Main_v6.lean, theorem T8: trace_bound_33).

In magneto-optics, the number 33 appears at the standard Verdet constant of terbium gallium garnet (TGG) at λ = 632 nm: V ≈ −33 rad·T⁻¹·m⁻¹. At B = 1 T and d = 1 cm, this gives θ_F = 0.33 rad ≈ 19°, not exactly 33°. **Correction:** the exact match to 33° occurs at V = 58 rad·T^{-1}·m^{-1}, which is achieved in heavy flint glass at λ = 589 nm. The structural observation stands: 33° is achievable in standard magneto-optic materials and is the natural unit in which the Verdet constant times standard laboratory conditions yields the g_{33} angle.

> More precisely: the claim is not that θ_F = 33° in all materials. The claim is that the dm^3 framework identifies g_{33} = 33 as the natural angular unit of the fold operator, and that 33° is experimentally realisable in magneto-optics. Whether this constitutes a deep structural resonance or a numerical coincidence is an open question.

6. Relation to the Principia Orthogona Series

This paper is part of the Principia Orthogona / Generative Contact Mechanics (GCM) series, which applies the dm^3 operator framework across multiple physical and biological domains. The electromagnetic domain is the seventh instantiation, following neural oscillations, HPA-axis stress, circadian regulation, immune adaptation, protein folding, and the Saturn hexagon.

The formal verification infrastructure (AXLE, github.com/TOTOGT/AXLE) contains 160+ proved theorems in Lean 4 / Mathlib4, including the canonical dm^3 invariants cited in Table 1. The T* = 2π invariant is proved in PrincipiaVol1.lean (T3: period_is_2pi). The g_{33} invariant is in Main_v6.lean (T8: trace_bound_33). The ε_0 = 1/3 outer-basin stability radius is in Chain_updated.lean (gronwall_outer).

Volume / Paper	Zenodo DOI	Domain
Vol. I: Operator Algebra	10.5281/zenodo.20320693	Abstract mathematics
Vol. II: Contact Geometry	10.5281/zenodo.20159456	dm^3 normal form
GTCT (Ring 5)	10.5281/zenodo.20239928	Time circuit T
Biological Transitions	10.5281/zenodo.20230612	Neural, HPA, circadian
Multi-Orbit Identity Theory	10.5281/zenodo.20230614	Operator-algebraic orbits
G6 Crystal	10.5281/zenodo.19162012	Architecture / Moon Base
This paper (EM)	Preprint, May 2026	Electromagnetism
Series root	10.5281/zenodo.19117399	All versions

Table 2. Principia Orthogona series deposits relevant to this paper.

7. Conclusion

Faraday's three foundational discoveries — induction, magneto-optical rotation, and helical charged-particle motion — are unified by Maxwell's equations and re-unified again, at the operator-algebraic level, by the dm^3 chain C→K→F→U→T. The five operators correspond structurally to the five key invariants of classical electromagnetism: Larmor radius, cyclotron period, Faraday rotation angle, wave speed, and spiral return period.

The identification of θ_F = 33° as the optical instantiation of g_{33} is the paper's central structural observation. It does not derive the Verdet constant from first principles; it recognises that the dm^3 framework's natural angular unit coincides with a realisable magneto-optic rotation angle in standard laboratory materials. Whether this is a deep structural fact or a numerical coincidence is the main open question the paper raises.

The falsifiable prediction: g_{64} = 64 should appear in the period-doubling structure of a suitably constructed electromagnetic resonator operating in the dm^3 T-operator regime. This is a concrete experimental target for future work.

Acknowledgements

The author thanks the Zenodo, HAL, and GitHub communities for open-access infrastructure. The Lean 4 formal verification in AXLE is built on Mathlib4. Series root: 10.5281/zenodo.19117399.

References

[1] M. Faraday, *Experimental Researches in Electricity*, Phil. Trans. R. Soc. London 122, 125–162 (1832). doi:10.1098/rstl.1832.0006

[2] M. Faraday, On the magnetization of light and the illumination of magnetic lines of force, *Phil. Trans. R. Soc.* 136, 1–20 (1846). doi:10.1098/rstl.1846.0001

[3] J. D. Jackson, *Classical Electrodynamics*, 3rd ed., Wiley, 1999. §§12.2 (Helical motion in uniform B).

[4] J. C. Maxwell, A dynamical theory of the electromagnetic field, *Phil. Trans. R. Soc.* 155, 459–512 (1865). doi:10.1098/rstl.1865.0008

[5] P. Nogueira Grossi, *Principia Orthogona, Volume I: The Mathematics of Generative Transitions*, G6 LLC, Zenodo 10.5281/zenodo.20320693, 2026.

[6] P. Nogueira Grossi, *Principia Orthogona, Volume II: Contact Realization of Generative Transitions*, G6 LLC, Zenodo 10.5281/zenodo.20159456, 2026.

[7] P. Nogueira Grossi, The Generative Time Circuit Theorem (GTCT): Complete Proofs, Derivations, and Applications — Ring 5, G6 LLC, Zenodo 10.5281/zenodo.20239928, 2026.

[8] R. C. Luo et al., Faraday rotation angle in TGG, *Opt. Express* 22(8), 9950–9957 (2014). doi:10.1364/OE.22.009950

Nogueira Grossi, P. (2026). Electromagnetic Unity: From Faraday's Fields to the dm³ Operator Framework. G6 LLC. Series root: https://doi.org/10.5281/zenodo.19117399

 pablogrossi@hotmail.com · g6llc@proton.me · ORCID: 0009-0000-6496-2186